Wavefolding: Modulation of Adjustable Symmetry in Sawtooth and Triangular Waveforms

ABSTRACT

The Pulse-Width Modulation (PWM) technique has been used to generate varying timbres of odd-harmonic spectra from early on in voltage controlled analog synthesis history. Methods for controlling the symmetry of a triangle-to-sawtooth wave have also been devised. This paper discusses a family of objects and techniques for piecewise waveform manipulation that may be modulated at audio rate, comparing the results with analog equivalents, and looking specifically at the implications of modulator phase and subtle deviations from integer carrier-to-modulator ratios, and fine deviations from these, on adjustable-symmetry sawtooth waves.

1. INTRODUCTION

The sawtooth or ramp wave is a fundamental element in subtractive synthesis, since it contains both odd and even harmonics of the fundamental frequency. It's slightly dull cousin, the triangle wave, has weak overtones of odd harmonics and sounds much like a digital approximation of a sine wave. Both have their uses in synthesis, but it is possible in both analog and digital domains to generate waveforms that can be modulated between sawtooth and triangle. Some digital synthesis methods have used this principle particularly since the transformation from a sawtooth wave into a triangle wave creates a reduction in harmonic richness similar (but not the same as) subtractive filters. Historically, Casio's ill-fated VZ series of synthesizers in the 1980s used a method called IPD or Interactive Phase Distortion, based on the transformation of waveforms through progressively sharper sawtooth shapes. Software glitches with the interface along with bad commercial timing (the Korg M1 released at the same time, which also had a sequencer and drums) led to the withdrawal of Casio from the pro-audio market.

With computer synthesis it is a simple procedure to create an algorithm that generates adjustable symmetry sawtooth-to-triangle waves that may be modulated at audio frequencies. Empirical research into harmonic spectra of such modulations reveals a slightly more complex morphology of spectra than would be devised using subtractive methods, and the application of single frequency modulation (sine-wave modulation) of the waveform results in complex timbre transformations over time, highly dependent on phase ratios between carrier and modulator, and a temporal morphology that reflects the characteristic shape of the sawtooth wave itself.

2. THE WAVEFOLDER~ OBJECT

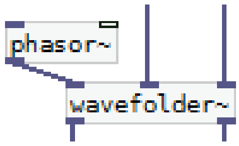

Figure 1. The wavefolder~ object generates variable asymmetry sawtooth/triangle waves from a phasor~ (ramp) input.

The implementation of an algorithm for converting a ramp wave into a triangle wave or inverse ramp is relatively simple. This was initially accomplished as a Pure Data[1] (Pd) patch using the sigpack~ library of objects[1]. More recently this has been created as an external for Pd, along with the wavestretcher~ object. This has simplified the process of converting a ramp from a phasor~ object into an adjustable-symmetry waveform, and opened-up the possibility of audio frequency modulation of the waveform symmetry.

The principle is simple. With a ramp waveform from 0 to 1, a threshold is set between 0 and 1. Sample-by-sample the output is given by:

$$O = IF(R > T; R(1/T); 1-((R-T)*(1/(1-T)) \quad (1)$$

where O = out sample, R = ramp input and T = threshold. Divide-by-zero errors are eliminated in a separate function that prevents R from arriving at precisely 0 or 1. This object can be found in the ekext library of Pd externals[2].

3. SPECTRAL CHARACTERISTICS

3.1 Frequency spectra at static symmetry settings

As the waveform is modulated between a setting of 0 (symmetric triangle waveform) and 1 (asymmetric ramp waveform), peaks and troughs in the harmonic spectrum are developed (see figures 1-4). This was empirically tested in order to establish the relationship between the symmetry of the waveform and the resultant harmonic spectrum, in order to establish how the functional description of a sawtooth or ramp wave is affected by this process. It makes sense to define this relationship in terms of deviation from the sawtooth or ramp waveform toward the triangle, as there is a reciprocal relationship

[1] https://puredata.info/downloads/sigpack
[2] Latest versions can be downloaded from http://sharktracks.co.uk/html/software.html

between the troughs in the resultant spectra and the symmetry of the waveform. Furthermore, the reduction in the magnitude of even harmonics is not linear.

There is a modulation between the Fourier series of a sawtooth wave:

$$x_{saw}=\frac{1}{2}-\frac{1}{\pi}\sum_{k=1}^{\infty}\left(\frac{\sin(2\pi kft)}{k}\right) \quad (2)$$

and that of a triangle wave:

$$x_{tri}=\frac{8}{\pi^2}\sum_{k=0}^{\infty}\left(\frac{\sin(2\pi(2k+1)ft)}{(2k+1)^2}\right) \quad (3)$$

As can be seen from the figures below, the modulation of the magnitudes of harmonics closely resembles a cosine function of the magnitudes based on the harmonic number, starting at infinity for the ideal saw and starting at harmonic 2 for the triangle. Given that, in additive synthesis both waveforms' harmonics are alternately opposite in phase to the previous harmonic (1, -2, 3, -4 etc and 1, -3, 5, -7 etc) there are clues to how the combination of additive sine elements with different phase relationships may result in the spectra observed below. An exponential relationship between the linear asymmetry and the position of the first trough in the spectrum is observed, and the interval in harmonics until the next of these, such that a triangle wave has an absence of even harmonics (2, 4, 6, 8...interval=2) and figures for alternative symmetry settings as shown in the table and graphical figures below:

Asymmetry	Interval
0 (triangle)	2
0.5	4
0.75	8
0.875	16
0.9325	32
1 (sawtooth)	Nyquist (SR/2)

Table 1. Asymmetry settings and their correspondent troughs in the harmonic spectrum.

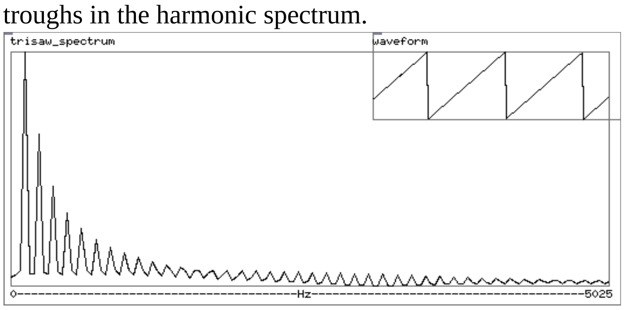

Figure 2. Spectrum and waveform at symmetry setting 1 (ramp waveform).

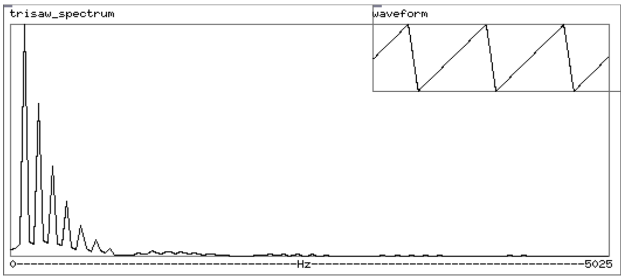

Figure 3. Symmetry setting 0.75.

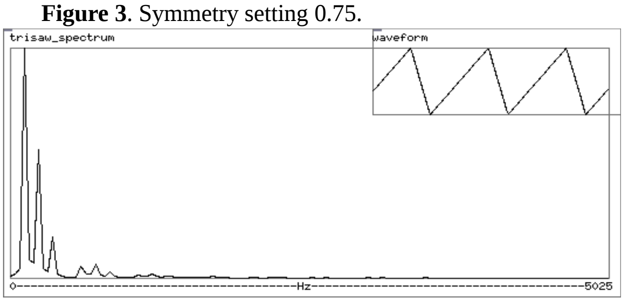

Figure 4. Spectrum and waveform at symmetry setting 0.5.

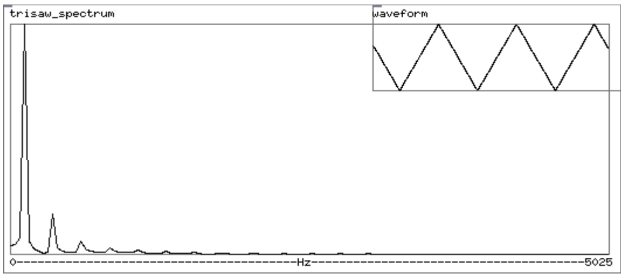

Figure 5. Spectrum and waveform at symmetry setting 0 (triangle waveform).

4. AUDIO FREQUENCY MODULATION OF THE WAVESHAPE

The shape of the wavefolder~ output is controllable at audio rate with limits of -1 (saw down) and 1 (saw up) with a setting of 0 representing the triangle waveform. The relationships between the phase of the modulation signal (in this case a simple sinusoidal waveform) and the phase of the asymmetry modulation are important to the resulting timbre. With a modulating sine function at the same frequency, at 270º there are more corners to the waveform, and more high-frequency harmonics are generated (fig. 7), whereas at 90º between trisaw and sine the waveform is more like a distended triangle wave and the harmonic spectrum is less bright (fig. 6).

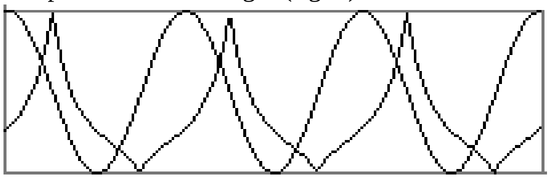

Figure 6. Superimposed waveforms of modulator and resultant waveform at modulator phase = 270º with respect to the tri/saw wave.

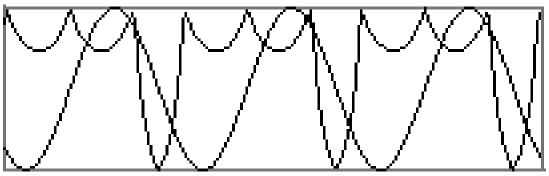

Figure 7. Superimposed waveforms of modulator and resultant waveform at phase = 90º.

4.1 Spectro-Morphology at Detuned Modulation Frequencies

Thus far this paper has considered static waveforms, and there is no single result here that cannot be achieved by a wavetable method of synthesis. But this method begins to yield more interesting results as the modulation waveform is detuned from integer-multiples of the asymmetric modulated waveform. The phase relationship discussed above is continually changing, and this results in morphological transitions between very bright, harsh-sounding timbres and softer timbres.

At a positive detuning away from the frequency of the tri/saw wave, the sweep is from bright-to-soft with a plateau at the brightest point, repeating at a rate equivalent to the difference in frequency between the carrier (tri/saw waveform) and the modulator (sine). The inverse is true at a negative detuning, that is the sweep in timbre is from soft-to-bright. More complex timbres are achieved with simple non-integer ratios (1.5, 0.75 etc) giving inharmonic timbres but with a degree of tonality. Just as with frequency modulation synthesis, the more complex the integer ratio of the carrier to the modulator, the more inharmonic the timbre produced.

Furthermore, since the sweep in brightness is a rhythmic effect, this can be controlled mathematically to be consistent across all integer-ratio carrier-to-modulator values, and an object has been created to facilitate this, which will be demonstrated at the conference and made available on the author's website.

4.2 Pulse-Width Modulations of the Modulated Asymmetric Waveform

The wavefolder~ object has an extra inlet and outlet at audio rate allowing for the modulated waveform to have a process of pulse-width modulation applied to it. Since the waveform shapes of a modulated asymmetric waveform are geometrically complex, a set of timbres are available from the object that are more varied than those of traditional PWM. When this is combined with the detuning of the modulator discussed above, the timbre evolution of the asymmetric waveform is transferred to the pulse waveform with the potential for modulations of the PWM threshold to create further evolutions in timbre.

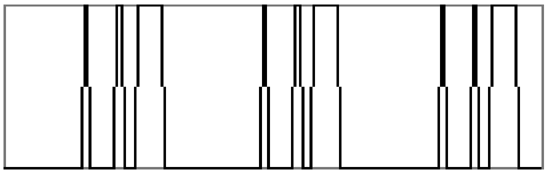

Figure 8. An example of the pulse-width modulated output from wavefolder~.

5. MORE PIECEWISE MANIPULATION

5.1 Wavestretcher~

A second object uses a similar approach the the wavefolder~ by taking a breakpoint (threshold) and manipulating the geometric angle of the waveform differently depending on which side of the threshold it is. It is useful to think of this as a complementary function to the previous object. While the wavefolder~ modulates from a sawtooth input (from phasor~) towards a triangle waveform using a breakpoint-based algorithm, wavestretcher modulates from the sawtooth (or any input waveform for that matter) towards pulse-train-style waveforms as shown below.

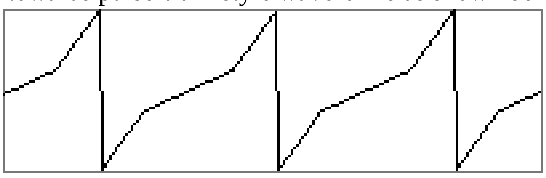

Figure 9. Stretched sawtooth waveform at breakpoint = 0 (middle of absolute value) and stretch factor at -0.5.

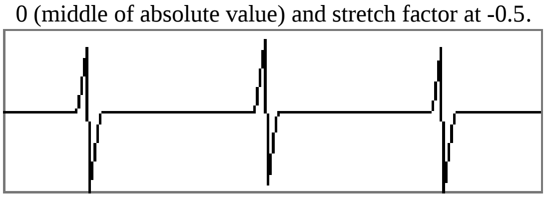

Figure 10. With the same sawtooth input, breakpoint = -0.75, stretch factor = -1.

Positive values of the stretch factor allow the modulation between triangular or sawtooth waveforms through trapezoidal waveforms until a square wave or clipped sawtooth waveform results.

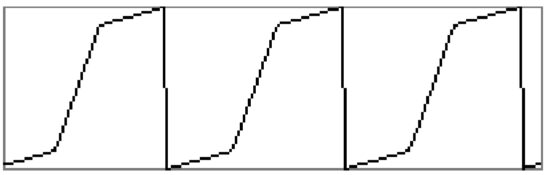

Figure 11. With the same sawtooth input, breakpoint = -0.5, stretch factor = 0.7.

The use of both objects, with the output of wavefolder~ feeding into wavestretcher~ affords a situation where a large repertoire of complex timbres may be generate using a highly compact, efficient structure. It is possible to emulate timbres of subtractive synthesis without the use of filters, but with a greater degree of flexibility in terms of timbre control[3].

[3] It must be stated that there is no way to reproduce high-Q resonant peaks without the use of audio filters in this system, although a phase-distortion-type equivalent may involve added resonant circuits to one portion of the waveform.

6. ANALOG REALIZATION

6.1 Sawtooth to Triangle Wave Modulation

There are some implementations of this idea available on synth-DIY sites by Hoshuyama [2]. Tillmans [3] and Gratz [4]. All of these articles observed by this author come with the caveat that they are "untested" e.g.[2] although this seems unlikely given the knowledge and experience of those contributing this knowledge, since such features exist in Moog and MFB synthesizers (Moog Voyager, MFB Dominion) and it would be naive to assume that the potential of these systems was overlooked[4], especially given the ubiquity of PWM in commercially successful forms of popular electronic music and electronic dance music (EDM) over the past four decades. However only the Moog Voyager XL appears to offer a fully patchable (and hence audio rate) modulation of the waveform shape.

Of particular interest for its simplicity is the design and article by Don Tillmans [2], published in 2000 and revised in 2002, providing a simple circuit for analog waveshaping of a sawtooth wave using two operational transconductance amplifiers. A certain amount is left to the circuit-builder to figure out in this article. As the original circuit uses hard-to-find CA3280 chips[5] efforts are ongoing to adapt the circuit to use a readily available LM13700 dual operational transconductance amplifier (OTA) integrated circuit (IC), although a new solution is detailed below, based on the wavefolder~ algorithm.

The analog circuit designed by Don Tillmans (cited by Gratz[3]) uses an equivalent equation to that in the digital object wavefolder~ expressed in a form more mathematically elegant than the coding algorithm expressed in part 2 above, thus:

$$\frac{1}{1+e^{x}}+\frac{1}{1+e^{-x}}=1 \tag{4}$$

where x is equivalent to the control voltage in the analog circuit, and the threshold/breakpoint value in the digital algorithm of wavefolder~. This accurately reflects the exponential relationship between the wavefolder~ threshold value and the harmonic modulations detailed in table 1, and figures 2-5.

Given that an exponential multiplication of a signal is the reciprocal of a division by a linear increase or decrease of the denominator, an alternative method can be devised for an analog circuit using the principles of the original wavefolder~ algorithm. An analog switch IC replaces the IF statements, and differential amplifiers are used to generate reciprocal control voltages for a voltage-controlled amplifier against a reference voltage. Both the input ramp wave and its inverted counterpart are switched alternately using a comparator, along with the control voltages to an exponential converter into an OTA. A single OTA may be used, and in this realization it is a CA3080 – the chip designed by RCA that was vital to the creation of early voltage-controlled synthesizers (the RCA Mk1 and Mk2) which, as the footnote below shows is now available again, albeit in lots of 100 ICs.

This circuit should perform in exactly the same way as the digital algorithm, with a threshold voltage determining the asymmetry of the resultant waveform. Effectively though, every analog implementation of this principle, from the low-frequency oscillator of the Korg MS20 to the switched OTA concept described above, uses the same principle of threshold-switching the separately amplified non-inverted and inverted portions of the ramp waveform on either side of the switching threshold.

[4] These articles are all over a decade old at the time of publication.

[5] These are now being manufactured by Rochester Electronics: http://www.rocelec.com

7. CONCLUSIONS

This project was driven by curiosity into a way of generating complex timbres from simple means, and how pushing methods from analog experimentation by synthesis enthusiasts into the digital domain may open new approaches (asymmetry modulation) to timbre modulation of basic synthesis waveforms. The conceptual process of development for the analog circuit was realized by understanding that through some lateral transposition of the principles of the digital implementation, an analog realization could be created based on the same principles as the digital object. A conceptual loop can be observed where code-based digital methods and analog electronics can be created in parallel, and where understanding from one branch of electronic music can be adapted to function using the same principles in another.

Acknowledgments

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8. REFERENCES

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[3] Tillmans, D, *Voltage Controlled Duty Cycle Sawtooth Circuit,* `www.till.com`, 1999, 2002.

[4] Gratz, A, *Triangle / Sawtooth VCO with voltage controlled continously variable symmetry,* `http://synth.stromeko.net/diy/SawWM.pdf`, 2006.